UK Senior

Mathematical Challenges

2011 to 2017

Bromsgrove International School Thailand

Senior Mathematical Challenge

Organised by the United Kingdom Mathematics Trust

supported by Institute and Faculty of Actuaries

7 November 2017

1. One of the following numbers is prime. Which is it?

A $2017 - 2$ B $2017 - 1$ C 2017 D $2017 + 1$ E $2017 + 2$

2. Last year, an earthworm from Wigan named Dave wriggled into the record books as the largest found in the UK. Dave was 40 cm long and had a mass of 26 g.

What was Dave's mass per unit length?

A 0.6 g/cm B 0.65 g/cm C 0.75 g/cm D 1.6 g/cm
E 1.75 g/cm

3. The five integers 2, 5, 6, 9, 14 are arranged into a different order. In the new arrangement, the sum of the first three integers is equal to the sum of the last three integers.

What is the middle number in the new arrangement?

A 2 B 5 C 6 D 9 E 14

4. Which of the following is equal to $2017 - \dfrac{1}{2017}$?

A $\dfrac{2017^2}{2016}$ B $\dfrac{2016}{2017}$ C $\dfrac{2018}{2017}$ D $\dfrac{4059}{2017}$

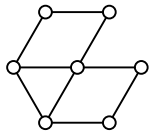
E $\dfrac{2018 \times 2016}{2017}$

5. One light-year is nearly 6×10^{12} miles. In 2016, the Hubble Space Telescope set a new cosmic record, observing a galaxy 13.4 thousand million light-years away.

Roughly how many miles is that?

A 8×10^{20} B 8×10^{21} C 8×10^{22} D 8×10^{23} E 8×10^{24}

6. The circles in the diagram are to be coloured so that any two circles connected by a line segment have different colours.



What is the smallest number of colours required?

A 2 B 3 C 4 D 5 E 6

7. The positive integer k satisfies the equation $\sqrt{2} + \sqrt{8} + \sqrt{18} = \sqrt{k}$.

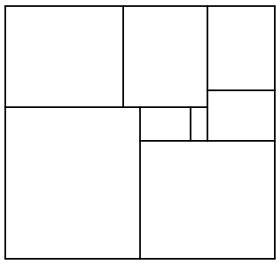
What is the value of k?

A 28 B 36 C 72 D 128 E 288

8. When evaluated, which of the following is not an integer?

A 1^{-1} B $4^{-\frac{1}{2}}$ C 6^0 D $8^{\frac{2}{3}}$ E $16^{\frac{3}{4}}$

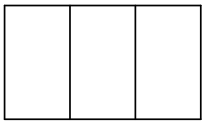
9. The diagram shows an $n\times(n+1)$ rectangle tiled with $k\times(k+1)$ rectangles, where n and k are integers and k takes each value from 1 to 8 inclusive.



What is the value of n?

A 16 B 15 C 14 D 13 E 12

10. A rectangle is divided into three smaller congruent rectangles as shown.



Each smaller rectangle is similar to the large rectangle.

In each of the four rectangles, what is the ratio of the length of a longer side to that of a shorter side?

A $2\sqrt{3} : 1$ B $3 : 1$ C $2 : 1$ D $\sqrt{3} : 1$ E $\sqrt{2} : 1$

11. The teenagers Sam and Jo notice the following facts about their ages:

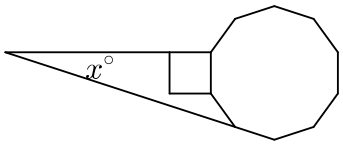
The difference between the squares of their ages is four times the sum of their ages.

The sum of their ages is eight times the difference between their ages.

What is the age of the older of the two?

A 15 B 16 C 17 D 18 E 19

12. The diagram shows a square and a regular decagon that share an edge. One side of the square is extended to meet an extended edge of the decagon.



What is the value of x?

A 15 B 18 C 21 D 24 E 27

13. Isobel: "Josh is innocent" Genotan: "Tegan is guilty"

Josh: "Genotan is guilty" Tegan: "Isobel is innocent"

Only the guilty person is lying; all the others are telling the truth.

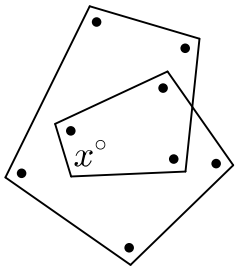
Who is guilty?

A Isobel B Josh C Genotan D Tegan
E More information required

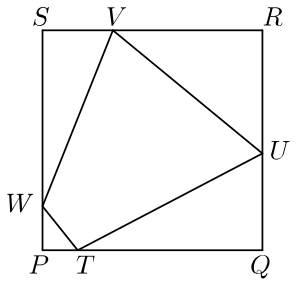
14. In the diagram all the angles marked • are equal in size to the angle marked $x°$.

What is the value of x?

A 100 B 105 C 110 D 115 E 120



15. The diagram shows a square $PQRS$. Points T, U, V and W lie on the edges of the square, as shown, such that $PT = 1$, $QU = 2$, $RV = 3$ and $SW = 4$.



The area of $TUVW$ is half that of $PQRS$.

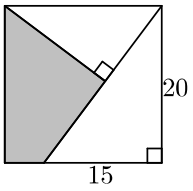
What is the length of PQ?

A 5 B 6 C 7 D 8 E 9

16. The diagram shows two right-angled triangles inside a square. The perpendicular edges of the larger triangle have lengths 15 and 20.

What is the area of the shaded quadrilateral?

A 142 B 146 C 150 D 154 E 158



17. Amy, Beth and Claire each has some sweets. Amy gives one third of her sweets to Beth. Beth gives one third of all the sweets she now has to Claire. Then Claire gives one third of all the sweets she now has to Amy. All the girls end up having the same number of sweets.

Claire begins with 40 sweets.

How many sweets does Beth have originally?

A 20 B 30 C 40 D 50 E 60

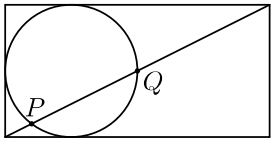
18. The arithmetic mean, A, of any two positive numbers x and y is defined to be $A = \frac{1}{2}(x+y)$ and their geometric mean, G, is defined to be $G = \sqrt{xy}$.

For two particular values x and y, with $x > y$, the ratio $A : G = 5 : 4$.

For these values of x and y, what is the ratio $x : y$?

A $5 : 4$ B $2 : 1$ C $5 : 2$ D $7 : 2$ E $4 : 1$

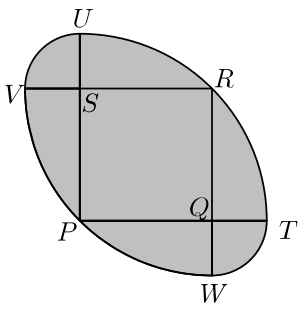
19. The diagram shows a circle of radius 1 touching three sides of a 2×4 rectangle. A diagonal of the rectangle intersects the circle at P and Q, as shown.



What is the length of the chord PQ?

A $\sqrt{5}$ B $\dfrac{4}{\sqrt{5}}$ C $\sqrt{5} - \dfrac{2}{\sqrt{5}}$ D $\dfrac{5\sqrt{5}}{6}$ E 2

20. The diagram shows a square $PQRS$ with edges of length 1, and four arcs, each of which is a quarter of a circle. Arc TRU has centre P; arc VPW has centre R; arc UV has centre S; and arc WT has centre Q.



What is the length of the perimeter of the shaded region?

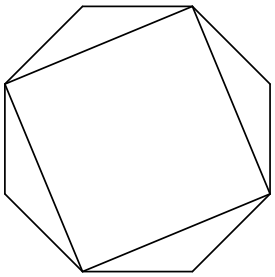
A 6 B $(2\sqrt{2}-1)\pi$ C $(\sqrt{2}-\frac{1}{2})\pi$

D 2π E $(3\sqrt{2}-2)\pi$

21. How many pairs (x, y) of positive integers satisfy the equation $4^x = y^2 + 15$?

A 0 B 1 C 2 D 4
E an infinite number

22. The diagram shows a regular octagon and a square formed by drawing four diagonals of the octagon. The edges of the square have length 1.



What is the area of the octagon?

A $\frac{\sqrt{6}}{2}$ B $\frac{4}{3}$ C $\frac{7}{5}$ D $\sqrt{2}$ E $\frac{3}{2}$

23. The parabola with equation $y = x^2$ is reflected in the line with equation $y = x + 2$.

Which of the following is the equation of the reflected parabola?

A $x = y^2 + 4y + 2$ B $x = y^2 + 4y - 2$ C $x = y^2 - 4y + 2$

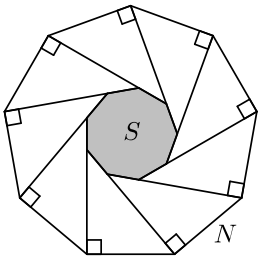
D $x = y^2 - 4y - 2$ E $x = y^2 + 2$

24. There is a set of straight lines in the plane such that each line intersects exactly ten others.

Which of the following could not be the number of lines in that set?

A 11 B 12 C 15 D 16 E 20

25. The diagram shows a regular nonagon N. Moving clockwise around N, at each vertex a line segment is drawn perpendicular to the preceding edge. This produces a smaller nonagon S, shown shaded.



What fraction of the area of N is the area of S?

A $\dfrac{1-\cos 40^\circ}{1+\cos 40^\circ}$ B $\dfrac{\cos 40^\circ}{1+\cos 40^\circ}$ C $\dfrac{\sin 40^\circ}{1+\sin 40^\circ}$

D $\dfrac{1-\sin 40^\circ}{1+\sin 40^\circ}$ E $\dfrac{1}{9}$

UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 8 November 2016

Organised by the **United Kingdom Mathematics Trust**

and supported by

Institute and Faculty of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
3. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
4. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. **Scoring rules**: all candidates start out with 25 marks;

   0 marks are awarded for each question left unanswered;

   4 marks are awarded for each correct answer;

   1 mark is deducted for each incorrect answer.
7. **Guessing**: Remember that there is a penalty for incorrect answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

http://www.ukmt.org.uk

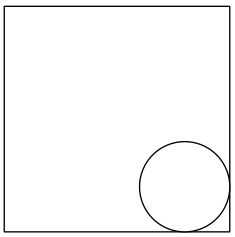
1. How many times does the digit 9 appear in the answer to $987\,654\,321 \times 9$?

A 0 B 1 C 5 D 8 E 9

2. On a Monday, all prices in Isla's shop are 10 % more than normal. On Friday all prices in Isla's shop are 10 % less than normal. James bought a book on Monday for £5.50. What would be the price of another copy of this book on Friday?

A £5.50 B £5.00 C £4.95 D £4.50 E £4.40

3. The diagram shows a circle with radius 1 that rolls without slipping around the inside of a square with sides of length 5.



The circle rolls once around the square, returning to its starting point.

What distance does the centre of the circle travel?

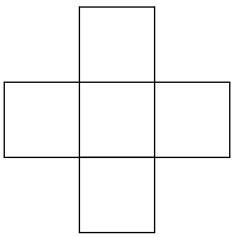
A $16 - 2\pi$ B 12 C $6 + \pi$ D $20 - 2\pi$ E 20

4. Alex draws a scalene triangle. One of the angles is 80°.

Which of the following could be the difference between the other two angles in Alex's triangle?

A 0° B 60° C 80° D 100° E 120°

5. All the digits 2, 3, 4, 5 and 6 are placed in the grid, one in each cell, to form two three-digit numbers that are squares.



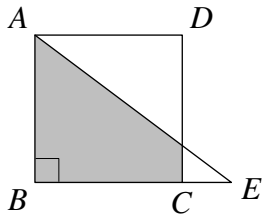
Which digit is placed in the centre of the grid?

A 2 B 3 C 4 D 5 E 6

6. The diagram shows a square $ABCD$ and a right-angled triangle ABE. The length of BC is 3. The length of BE is 4.

What is the area of the shaded region?

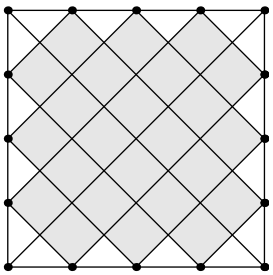
A $5\frac{1}{4}$ B $5\frac{3}{8}$ C $5\frac{1}{2}$ D $5\frac{5}{8}$ E $5\frac{3}{4}$



7. Which of these has the smallest value?

A 2016^{-1} B $2016^{-1/2}$ C 2016^{0} D $2016^{1/2}$ E 2016^{1}

8. Points are drawn on the sides of a square, dividing each side into n equal parts (so, in the example shown, $n = 4$). The points are joined in the manner indicated, to form several small squares (24 in the example, shown shaded) and some triangles.



How many small squares are formed when $n = 7$?

A 56 B 84 C 140 D 840
E 5040

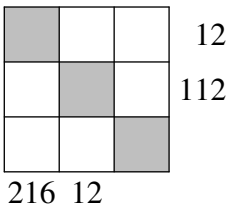
9. A square has vertices at (0,0), (1,0), (1,1) and (0,1). Graphs of the following equations are drawn on the same set of axes as the square.

$$x^2 + y^2 = 1, \qquad y = x + 1, \qquad y = -x^2 + 1, \qquad y = x, \qquad y = \frac{1}{x}$$

How many of the graphs pass through exactly two of the vertices of the square?

A 1 B 2 C 3 D 4 E 5

10. The digits from 1 to 9 are to be written in the nine cells of the 3×3 grid shown, one digit in each cell.



The product of the three digits in the first row is 12.

The product of the three digits in the second row is 112.

The product of the three digits in the first column is 216.

The product of the three digits in the second column is 12.

What is the product of the digits in the shaded cells?

A 24 B 30 C 36 D 48 E 140

11. In the grid below each of the blank squares and the square marked X are to be filled by the mean of the two numbers in its adjacent squares.

Which number should go in the square marked X?

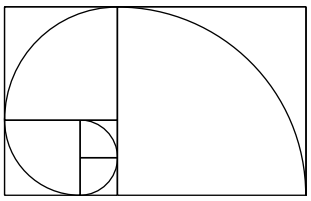
10			X		25

A 15 B 16 C 17 D 18 E 19

12. Which is the smallest square that has 2016 as a factor?

A 42^2 B 84^2 C 168^2 D 336^2 E 2016^2

13. Five square tiles are put together side by side. A quarter circle is drawn on each tile to make a continuous curve as shown. Each of the smallest squares has side-length 1.



What is the total length of the curve?

A 6π B 6.5π C 7π D 7.5π E 8π

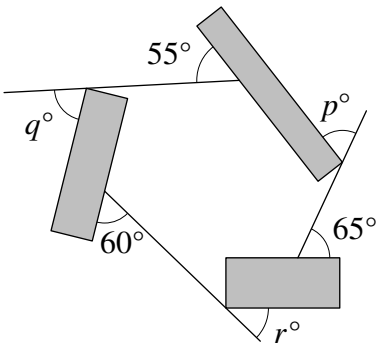
14. Which of the following values of the positive integer n is a counterexample to the statement: "If n is not prime then $n-2$ is not prime"?

A 6 B 11 C 27 D 33 E 51

15. The diagram shows three rectangles and three straight lines.

What is the value of $p + q + r$?

A 135 B 180 C 210 D 225
E 270



16. For which value of k is $\sqrt{2016} + \sqrt{56}$ equal to 14^k?

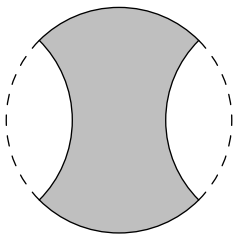
A $\frac{1}{2}$ B $\frac{3}{4}$ C $\frac{5}{4}$ D $\frac{3}{2}$ E $\frac{5}{2}$

17. Aaron has to choose a three-digit code for his bike lock. The digits can be chosen from 1 to 9. To help him remember them, Aaron chooses three different digits in increasing order, for example 278.

How many such codes can be chosen?

A 779 B 504 C 168 D 84 E 9

18. The circumference of a circle with radius 1 is divided into four equal arcs. Two of the arcs are 'turned over' as shown.



What is the area of the shaded region?

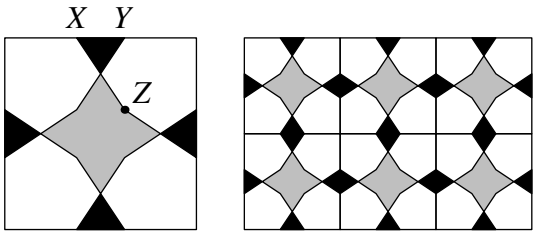
A 1 B $\sqrt{2}$ C $\frac{1}{2}\pi$ D $\sqrt{3}$ E 2

19. Let S be a set of five different positive integers, the largest of which is m. It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of S.

What is the smallest possible value of m?

A 2 B 4 C 9 D 11 E 12

20. Michael was walking in Marrakesh when he saw a tiling formed by tessellating the square tile as shown.

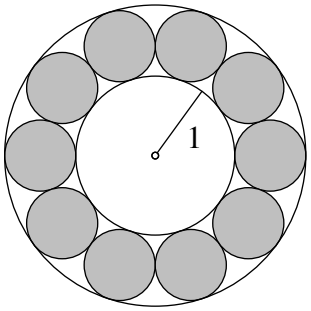


The tile has four lines of symmetry and the length of each side is 8 cm. The length of XY is 2 cm. The point Z is such that XZ is a straight line and YZ is parallel to the sides of the square.

What is the area of the central grey octagon?

A $6\,\text{cm}^2$ B $7\,\text{cm}^2$ C $8\,\text{cm}^2$ D $9\,\text{cm}^2$ E $10\,\text{cm}^2$

21. The diagram shows ten equal discs that lie between two concentric circles - an inner circle and an outer circle. Each disc touches two neighbouring discs and both circles. The inner circle has radius 1.



What is the radius of the *outer* circle?

A $2\tan 36°$ B $\dfrac{\sin 36°}{1-\sin 36°}$ C $\dfrac{1+\sin 18°}{1-\sin 18°}$

D $\dfrac{2}{\cos 18°}$ E $\dfrac{9}{5}$

22. Three friends make the following statements.

Ben says, "Exactly one of Dan and Cam is telling the truth."

Dan says, "Exactly one of Ben and Cam is telling the truth."

Cam says, "Neither Ben nor Dan is telling the truth."

Which of the three friends is lying?

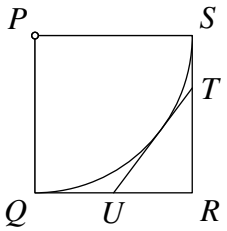
A Just Ben
B Just Dan
C Just Cam
D Each of Ben and Cam
E Each of Ben, Cam and Dan

23. A cuboid has sides of lengths 22, 2 and 10. It is contained within a sphere of the smallest possible radius.

What is the side-length of the largest cube that will fit inside the same sphere?

A 10 B 11 C 12 D 13 E 14

24. The diagram shows a square $PQRS$. The arc QS is a quarter circle. The point U is the midpoint of QR and the point T lies on SR. The line TU is a tangent to the arc QS.



What is the ratio of the length of TR to the length of UR?

A 3 : 2 B 4 : 3 C 5 : 4 D 7 : 6 E 9 : 8

25. Let n be the smallest integer for which $7n$ has 2016 digits.

What is the units digit of n?

A 0 B 1 C 4 D 6 E 8

Senior Mathematical Challenge

Thursday 5th November 2015

Organised by the United Kingdom Mathematics Trust

supported by Institute and Faculty of Actuaries

1. What is $2015^2 - 2016 \times 2014$?

A -2015 B -1 C 0 D 1 E 2015

2. What is the sum of all the solutions of the equation $6x = \dfrac{150}{x}$?

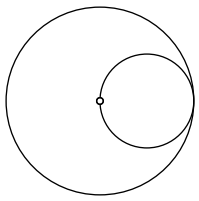
A 0 B 5 C 6 D 25 E 156

3. When Louise had her first car, 50 litres of petrol cost £40. When she filled up the other day, she noticed that 40 litres of petrol cost £50.

By approximately what percentage has the cost of petrol increased over this time?

A 50% B 56% C 67% D 75% E 80%

4. In the diagram, the smaller circle touches the larger circle and also passes through its centre.



What fraction of the area of the larger circle is outside the smaller circle?

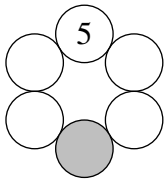
A $\frac{2}{3}$ B $\frac{3}{4}$ C $\frac{4}{5}$ D $\frac{5}{6}$ E $\frac{6}{7}$

5. The integer n is the mean of the three numbers 17, 23 and $2n$.

What is the sum of the digits of n?

A 4 B 5 C 6 D 7 E 8

6. The numbers 5, 6, 7, 8, 9, 10 are to be placed, one in each of the circles in the diagram, so that the sum of the numbers in each pair of touching circles is a prime number. The number 5 is placed in the top circle.



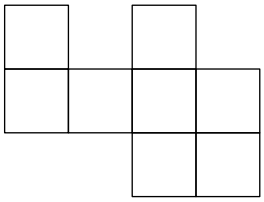
Which number is placed in the shaded circle?

A 6 B 7 C 8 D 9 E 10

7. Which of the following has the largest value?

A $\dfrac{(\frac{1}{2})}{(\frac{3}{4})}$ B $\dfrac{1}{\left(\dfrac{(\frac{2}{3})}{4}\right)}$ C $\dfrac{\left(\dfrac{(\frac{1}{2})}{3}\right)}{4}$ D $\dfrac{1}{\left(\dfrac{2}{(\frac{3}{4})}\right)}$ E $\dfrac{\left(\dfrac{1}{(\frac{2}{3})}\right)}{4}$

8. The diagram shows eight small squares. Six of these squares are to be shaded so that the shaded squares form the net of a cube.



In how many different ways can this be done?

A 10 B 8 C 7 D 6 E 4

9. Four different straight lines are drawn on a flat piece of paper. The number of points where two or more lines intersect is counted.

Which of the following could *not* be the number of such points?

A 1 B 2 C 3 D 4 E 5

10. The positive integer n is between 1 and 20. Milly adds up all the integers from 1 to n inclusive. Billy adds up all the integers from $n + 1$ to 20 inclusive. Their totals are the same.

What is the value of n?

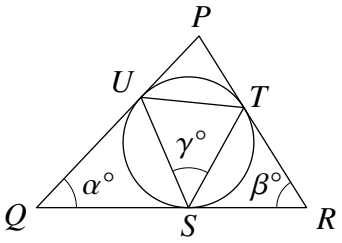
A 11 B 12 C 13 D 14 E 15

11. Rahid has a large number of cubic building blocks. Each block has sides of length 4 cm, 6 cm or 10 cm. Rahid makes little towers built from three blocks stacked on top of each other.

How many different heights of tower can he make?

A 6 B 8 C 9 D 12 E 27

12. A circle touches the sides of triangle PQR at the points S, T and U as shown. Also $\angle PQR = \alpha°$, $\angle PRQ = \beta°$ and $\angle TSU = \gamma°$.



Which of the following gives γ in terms of α and β?

A $\frac{1}{2}(\alpha + \beta)$ B $180 - \frac{1}{2}(\alpha + \beta)$
C $180 - (\alpha + \beta)$ D $\alpha + \beta$
E $\frac{1}{3}(\alpha + \beta)$

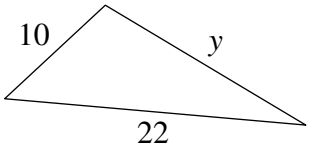
13. The Knave of Hearts tells only the truth on Mondays, Tuesdays, Wednesdays and Thursdays. He tells only lies on all the other days. The Knave of Diamonds tells only the truth on Fridays, Saturdays, Sundays and Mondays. He tells only lies on all the other days. On one day last week, they both said, "Yesterday I told lies."

On which day of the week was that?

A Sunday B Monday C Tuesday D Thursday E Friday

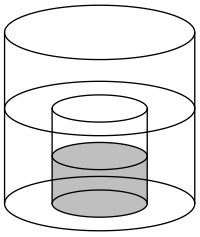
14. The triangle shown has an area of 88 square units.

What is the value of y?



A 17.6 B $2\sqrt{46}$ C $6\sqrt{10}$ D $13\sqrt{2}$
E $8\sqrt{5}$

15. Two vases are cylindrical in shape. The larger vase has diameter 20 cm. The smaller vase has diameter 10 cm and height 16 cm. The larger vase is partially filled with water. Then the empty smaller vase, with the open end at the top, is slowly pushed down into the water, which flows over its rim. When the smaller vase is pushed right down, it is half full of water.



What was the original depth of the water in the larger vase?

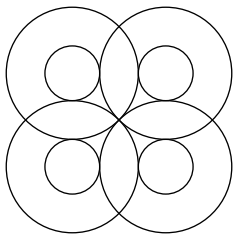
A 10 cm B 12 cm C 14 cm D 16 cm
E 18 cm

16. Fnargs are either red or blue and have 2, 3 or 4 heads. A group of six Fnargs consisting of one of each possible form is made to line up such that no immediate neighbours are the same colour nor have the same number of heads.

How many ways are there of lining them up from left to right?

A 12 B 24 C 60 D 120 E 720

17. The diagram shows eight circles of two different sizes. The circles are arranged in concentric pairs so that the centres form a square. Each larger circle touches one other larger circle and two smaller circles. The larger circles have radius 1.



What is the radius of each smaller circle?

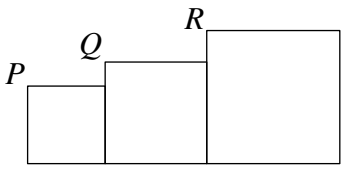
A $\frac{1}{3}$ B $\frac{2}{5}$ C $\sqrt{2}-1$ D $\frac{1}{2}$ E $\frac{1}{2}\sqrt{2}$

18. What is the largest integer k whose square k^2 is a factor of 10!?

[$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.]

A 6 B 256 C 360 D 720 E 5040

19. Three squares are arranged as shown so that their bases lie on a straight line. Also, the corners P, Q and R lie on a straight line. The middle square has sides that are 8 cm longer than the sides of the smallest square. The largest square has sides of length 50 cm. There are two possible values for the length (in cm) of the sides of the smallest square.



Which of the following are they?

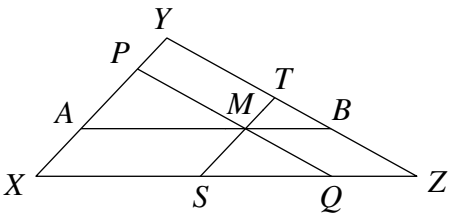
A 2, 32 B 4, 42 C 4, 34 D 32, 40 E 34, 42

20. A square ink pad has sides of length 1 cm. It is covered in black ink and carefully placed in the middle of a piece of white paper. The square pad is then rotated 180° about one of its corners so that all of the pad remains in contact with the paper throughout the turn. The pad is then removed from the paper.

What area of paper, in cm^2, is coloured black?

A $\pi + 2$ B $2\pi - 1$ C 4 D $2\pi - 2$ E $\pi + 1$

21. The diagram shows a triangle XYZ. The sides XY, YZ and XZ have lengths 2, 3 and 4 respectively. The lines AMB, PMQ and SMT are drawn parallel to the sides of triangle XYZ so that AP, QS and BT are of equal length.



What is the length of AP?

A $\frac{10}{11}$ B $\frac{11}{12}$ C $\frac{12}{13}$ D $\frac{13}{14}$ E $\frac{14}{15}$

22. Let $f(x) = x + \sqrt{x^2+1} + \dfrac{1}{x - \sqrt{x^2+1}}$.

What is the value of $f(2015)$?

A -1 B 0 C 1 D $\sqrt{2016}$ E 2015

23. Given four different non-zero digits, it is possible to form 24 different four-digit numbers containing each of these four digits.

What is the largest prime factor of the sum of the 24 numbers?

A 23 B 93 C 97 D 101 E 113

24. Peter has 25 cards, each printed with a different integer from 1 to 25. He wishes to place N cards in a single row so that the numbers on every adjacent pair of cards have a prime factor in common.

What is the largest value of N for which this is possible?

A 16 B 18 C 20 D 22 E 24

25. A function, defined on the set of positive integers, is such that $f(xy) = f(x) + f(y)$ for all x and y. It is known that $f(10) = 14$ and $f(40) = 20$.

What is the value of $f(500)$?

A 29 B 30 C 39 D 48 E 50

Senior Mathematical Challenge

Thursday 6 November 2014

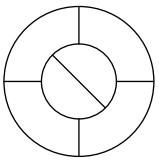
Organised by the United Kingdom Mathematics Trust

supported by Institute and Faculty of Actuaries

1. What is 98×102?

A 200 B 9016 C 9996 D 998 E 99 996

2. The diagram shows 6 regions. Each of the regions is to be painted a single colour, so that no two regions sharing an edge have the same colour.



What is the smallest number of colours required?

A 2 B 3 C 4 D 5 E 6

3. December 31st 1997 was a Wednesday.

How many Wednesdays were there in 1997?

A 12 B 51 C 52 D 53 E 365

4. After I had spent $\frac{1}{5}$ of my money and then spent $\frac{1}{4}$ of what was left, I had £15 remaining.

How much did I start with?

A £25 B £75 C £100 D £135 E £300

5. How many integers between 1 and 2014 are multiples of both 20 and 14?

A 7 B 10 C 14 D 20 E 28

6. In the addition sum shown, each of the letters T, H, I and S represents a non-zero digit.

$$\begin{array}{r} T\,H\,I\,S \\ +\quad\quad I\,S \\ \hline 2\,0\,1\,4 \end{array}$$

What is $T + H + I + S$?

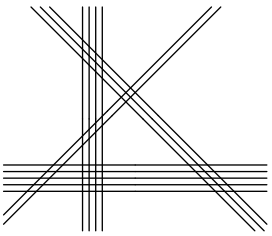
A 34 B 22 C 15 D 9 E 7

7. According to recent research, global sea levels could rise 36.8 cm by the year 2100 as a result of melting ice.

Roughly how many millimetres is that per year?

A 10 B 4 C 1 D 0.4 E 0.1

8. The diagram shows four sets of parallel lines, containing 2, 3, 4 and 5 lines respectively.



How many points of intersection are there?

A 54 B 63 C 71 D 95 E 196

9. Which of the following is divisible by 9?

A $10^{2014} + 5$ B $10^{2014} + 6$ C $10^{2014} + 7$ D $10^{2014} + 8$
E $10^{2014} + 9$

10. A rectangle has area $120\,\text{cm}^2$ and perimeter 46 cm.

Which of the following is the length of each of the diagonals?

A 15 cm B 16 cm C 17 cm D 18 cm E 19 cm

11. A Mersenne prime is a prime of the form $2^p - 1$, where p is also a prime.

One of the following is *not* a Mersenne prime. Which one is it?

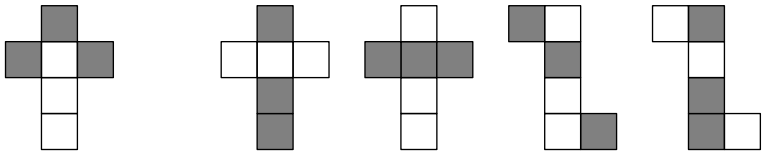
A $2^2 - 1$ B $2^3 - 1$ C $2^5 - 1$ D $2^7 - 1$ E $2^{11} - 1$

12. Karen has three times the number of cherries that Lionel has, and twice the number of cherries that Michael has. Michael has seven more cherries than Lionel.

How many cherries do Karen, Lionel and Michael have altogether?

A 12 B 42 C 60 D 77 E 84

13. Each of the five nets P, Q, R, S and T is made from six squares. Both sides of each square have the same colour. Net P is folded to form a cube.



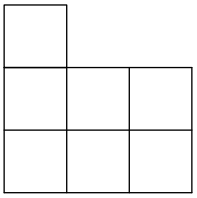
How many of the nets Q, R, S and T can be folded to produce a cube that looks the same as that produced by P?

A 0 B 1 C 2 D 3 E 4

14. Given that $\dfrac{3x+y}{x-3y} = -1$, what is the value of $\dfrac{x+3y}{3x-y}$?

A -1 B 2 C 4 D 5 E 7

15. The figure shown alongside is made from seven small squares. Some of these squares are to be shaded so that:

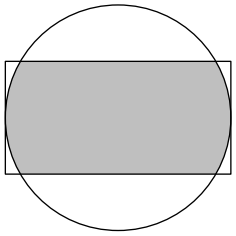


(i) at least two squares are shaded;
(ii) two squares meeting along an edge or at a corner are not both shaded.

How many ways are there to do this?

A 4 B 8 C 10 D 14 E 18

16. The diagram shows a rectangle measuring 6×12 and a circle.



The two shorter sides of the rectangle are tangents to the circle. The circle and rectangle have the same centre.

The region that lies inside both the rectangle and the circle is shaded. What is its area?

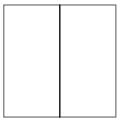
A $12\pi + 18\sqrt{3}$ B $24\pi - 3\sqrt{3}$ C $18\pi - 8\sqrt{3}$
D $18\pi + 12\sqrt{3}$ E $24\pi + 18\sqrt{3}$

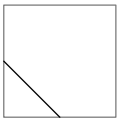
17. An oil tanker is 100 km due north of a cruise liner. The tanker sails SE at a speed of 20 kilometres per hour and the liner sails NW at a speed of 10 kilometres per hour.

What is the shortest distance between the two boats during the subsequent motion?

A 100 km B 80 km C $50\sqrt{2}$ km D 60 km E $33\frac{1}{3}$ km

18. Beatrix decorates the faces of a cube, whose edges have length 2. For each face, she either leaves it blank, or draws a single straight line on it. Every line joins the midpoints of two edges, either opposite or adjacent, as shown.

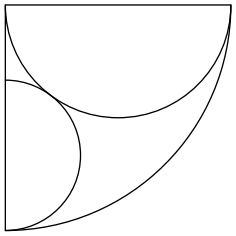




What is the length of the longest unbroken line that Beatrix can draw on the cube?

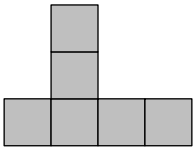
A 8 B $4+4\sqrt{2}$ C $6+3\sqrt{2}$ D $8+2\sqrt{2}$
E 12

19. The diagram shows a quadrant of radius 2, and two touching semicircles. The larger semicircle has radius 1. What is the radius of the smaller semicircle?



A $\frac{\pi}{6}$ B $\frac{\sqrt{3}}{2}$ C $\frac{1}{2}$ D $\frac{1}{\sqrt{3}}$ E $\frac{2}{3}$

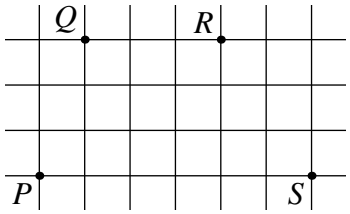
20. The diagram shows six squares with sides of length 2 placed edge-to-edge.



What is the radius of the smallest circle containing all six squares?

A $2\sqrt{5}$ B $2\sqrt{6}$ C 5 D $\sqrt{26}$ E $2\sqrt{7}$

21. Fiona wants to draw a 2-dimensional shape whose perimeter passes through all of the points P, Q, R and S on the grid of squares shown.



Which of the following can she draw?

(i) A circle
(ii) An equilateral triangle
(iii) A square

A only (i) and (ii) B only (ii) and (iii) C only (i) and (iii)
D all of (i), (ii) and (iii) E none of (i), (ii) and (iii)

22. A bag contains m blue and n yellow marbles. One marble is selected at random from the bag and its colour is noted. It is then returned to the bag along with k other marbles of the same colour. A second marble is now selected at random from the bag.

What is the probability that the second marble is blue?

A $\dfrac{m}{m+n}$ B $\dfrac{n}{m+n}$ C $\dfrac{m}{m+n+k}$ D $\dfrac{m+k}{m+n+k}$ E $\dfrac{m+n}{m+n+k}$

23. Which of the following have no real solutions?

(i) $2x < 2^x < x^2$ (ii) $x^2 < 2x < 2^x$ (iii) $2^x < x^2 < 2x$ (iv) $x^2 < 2^x < 2x$
(v) $2^x < 2x < x^2$ (vi) $2x < x^2 < 2^x$

A (i) and (iii) B (i) and (iv) C (ii) and (iv) D (ii) and (v)
E (iii) and (v)

24. Which of the following is smallest?

A $10 - 3\sqrt{11}$ B $8 - 3\sqrt{7}$ C $5 - 2\sqrt{6}$ D $9 - 4\sqrt{5}$
E $7 - 4\sqrt{3}$

25. Figure 1 shows a tile in the form of a trapezium, where $\alpha = 83\frac{1}{3}^\circ$. Several copies of the tile are placed together to form a symmetrical pattern, part of which is shown in Figure 2. The outer border of the complete pattern is a regular 'star polygon'. Figure 3 shows an example of a regular 'star polygon'.

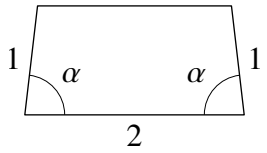


Figure 1

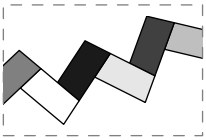


Figure 2

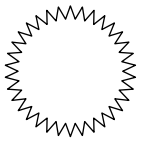


Figure 3

How many tiles are there in the complete pattern?

A 48 B 54 C 60 D 66 E 72

Senior Mathematical Challenge

Thursday 7 November 2013

Organised by the United Kingdom Mathematics Trust

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1. Which of these is the largest number?

A $2+0+1+3$ B $2\times 0+1+3$ C $2+0\times 1+3$ D $2+0+1\times 3$
E $2\times 0\times 1\times 3$

2. Little John claims he is 2 m 8 cm and 3 mm tall.

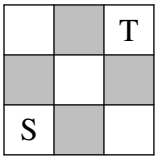
What is this height in metres?

A 2.83 m B 2.803 m C 2.083 m D 2.0803 m E 2.0083 m

3. What is the 'tens' digit of $2013^2 - 2013$?

A 0 B 1 C 4 D 5 E 6

4. A route on the 3×3 board shown consists of a number of steps. Each step is from one square to an adjacent square of a different colour.



How many different routes are there from square S to square T which pass through every other square exactly once?

A 0 B 1 C 2 D 3 E 4

6. Rebecca went swimming yesterday. After a while she had covered one fifth of her intended distance. After swimming six more lengths of the pool, she had covered one quarter of her intended distance.

How many lengths of the pool did she intend to complete?

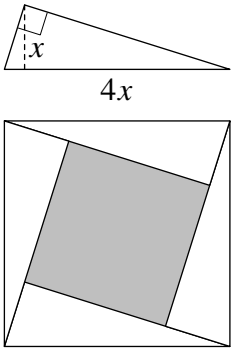
A 40 B 72 C 80 D 100 E 120

7. In a 'ninety nine' shop all items cost a number of pounds and 99 pence. Susanna spent £65.76.

How many items did she buy?

A 23 B 24 C 65 D 66 E 76

8. The right-angled triangle shown has a base which is 4 times its height. Four such triangles are placed so that their hypotenuses form the boundary of a large square as shown.



What is the side length of the shaded square in the diagram?

A $2x$ B $2\sqrt{2}x$ C $3x$ D $2\sqrt{3}x$
E $\sqrt{15}x$

9. According to a headline 'Glaciers in the French Alps have lost a quarter of their area in the past 40 years'.

What is the approximate percentage reduction in the length of the side of a square when it loses one quarter of its area, thereby becoming a smaller square?

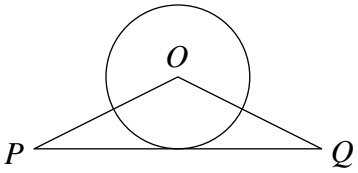
A 13% B 25% C 38% D 50% E 65%

10. Frank's teacher asks him to write down five integers such that the median is one more than the mean, and the mode is one greater than the median. Frank is also told that the median is 10.

What is the smallest possible integer that he could include in his list?

A 3 B 4 C 5 D 6 E 7

11. The diagram shows a circle with centre O and a triangle OPQ. Side PQ is a tangent to the circle. The area of the circle is equal to the area of the triangle.



What is the ratio of the length of PQ to the circumference of the circle?

A 1 : 1 B 2 : 3 C 2 : π D 3 : 2 E π : 2

12. As a special treat, Sammy is allowed to eat five sweets from his very large jar which contains many sweets of each of three flavours – Lemon, Orange and Strawberry. He wants to eat his five sweets in such a way that no two consecutive sweets have the same flavour.

In how many ways can he do this?

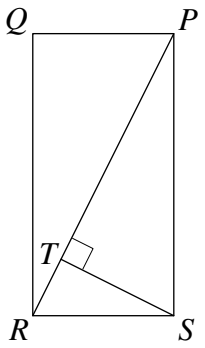
A 32 B 48 C 72 D 108 E 162

13. Two entrants in a school's sponsored run adopt different tactics. Angus walks for half the time and runs for the other half, whilst Bruce walks for half the distance and runs for the other half. Both competitors walk at 3 mph and run at 6 mph. Angus takes 40 minutes to complete the course.

How many minutes does Bruce take?

A 30 B 35 C 40 D 45 E 50

14. The diagram shows a rectangle $PQRS$ in which $PQ : QR = 1 : 2$. The point T on PR is such that ST is perpendicular to PR.



What is the ratio of the area of the triangle RST to the area of the rectangle $PQRS$?

A $1 : 4\sqrt{2}$ B $1 : 6$ C $1 : 8$ D $1 : 10$
E $1 : 12$

15. For how many positive integers n is $4^n - 1$ a prime number?

A 0 B 1 C 2 D 3
E infinitely many

16. Andrew states that every composite number of the form $8n + 3$, where n is an integer, has a prime factor of the same form.

Which of these numbers is an example showing that Andrew's statement is false?

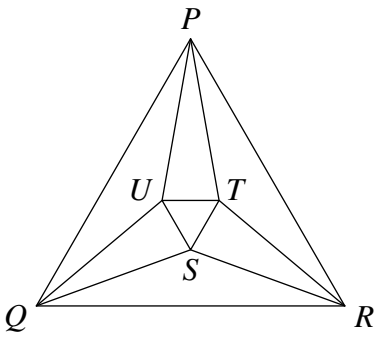
A 19 B 33 C 85 D 91 E 99

17. The equilateral triangle PQR has side length 1. The lines PT and PU trisect the angle RPQ, the lines RS and RT trisect the angle QRP and the lines QS and QU trisect the angle PQR.

What is the side length of the equilateral triangle STU?

A $\dfrac{\cos 80^\circ}{\cos 20^\circ}$ B $\frac{1}{3}\cos 20^\circ$ C $\cos^2 20^\circ$

D $\frac{1}{6}$ E $\cos 20^\circ \cos 80^\circ$

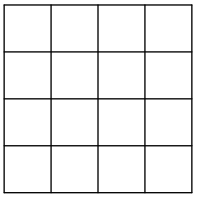


18. The numbers 2, 3, 12, 14, 15, 20, 21 may be divided into two sets so that the product of the numbers in each set is the same.

What is this product?

A 420 B 1260 C 2520 D 6720 E 6 350 400

19. The 16 small squares shown in the diagram each have a side length of 1 unit.



How many pairs of vertices are there in the diagram whose distance apart is an integer number of units?

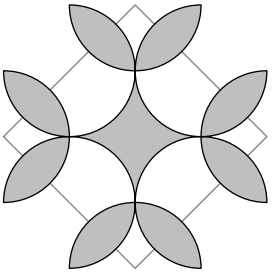
A 40 B 64 C 108 D 132 E 16

20. The ratio of two positive numbers equals the ratio of their sum to their difference.

What is this ratio?

A $(1 + \sqrt{3}) : 2$ B $\sqrt{2} : 1$ C $(1 + \sqrt{5}) : 2$ D $(2 + \sqrt{2}) : 1$
E $(1 + \sqrt{2}) : 1$

21. The shaded design shown in the diagram is made by drawing eight circular arcs, all with the same radius. The centres of four arcs are the vertices of the square; the centres of the four touching arcs are the midpoints of the sides of the square. The diagonals of the square have length 1.



What is the total length of the border of the shaded design?

A 2π B $\frac{5}{2}\pi$ C 3π D $\frac{7}{2}\pi$ E 4π

22. Consider numbers of the form $10n + 1$, where n is a positive integer. We shall call such a number 'grime' if it cannot be expressed as the product of two smaller numbers, possibly equal, both of which are of the form $10k + 1$, where k is a positive integer.

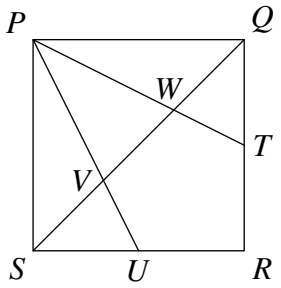
How many 'grime numbers' are there in the sequence $11, 21, 31, 41, \ldots, 981, 991$?

A 0 B 8 C 87 D 92 E 99

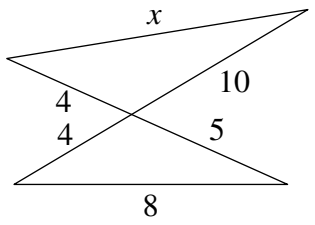
23. $PQRS$ is a square. The points T and U are the midpoints of QR and RS respectively. The line QS cuts PT and PU at W and V respectively.

What fraction of the total area of the square $PQRS$ is the area of the pentagon $RTWVU$?

A $\frac{1}{3}$ B $\frac{2}{5}$ C $\frac{3}{7}$ D $\frac{5}{12}$ E $\frac{4}{15}$



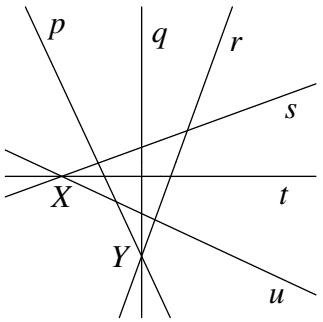
24. The diagram shows two straight lines PR and QS crossing at O.



What is the value of x?

A $7\sqrt{2}$ B $2\sqrt{29}$ C $14\sqrt{2}$
D $7\left(1+\sqrt{13}\right)$ E $9\sqrt{2}$

25. Challengeborough's underground train network consists of six lines p, q, r, s, t and u, as shown. Wherever two lines meet there is a station which enables passengers to change lines. On each line, each train stops at every station.



Jessica wants to travel from station X to station Y. She does not want to use any line more than once, nor return to station X after leaving it, nor leave station Y after reaching it.

How many different routes, satisfying these conditions, can she choose?

A 9 B 36 C 41 D 81 E 720

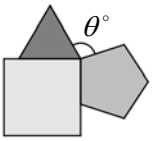
UK SENIOR MATHEMATICAL CHALLENGE

November 6th 2012

1. Which of the following cannot be written as the sum of two prime numbers?

A 5 B 7 C 9 D 10 E 11

2. The diagram shows an equilateral triangle, a square and a regular pentagon which all share a common vertex. What is the value of θ?



A 98 B 102 C 106 D 110 E 112

3. The price of my favourite soft drink has gone up in leaps and bounds over the past ten years. In four of those years it has leapt up by 5p each year, whilst in the other six years it has bounded up by 2p each year. The drink cost 70p in 2002. How much does it cost now?

A £0.77 B £0.90 C £0.92 D £1.02 E £1.05

4. According to one astronomer, there are one hundred thousand million galaxies in the universe, each containing one hundred thousand million stars. How many stars is that altogether?

A 10^{13} B 10^{22} C 10^{100} D 10^{120} E 10^{121}

5. All six digits of three 2-digit numbers are different. What is the largest possible sum of three such numbers?

A 237 B 246 C 255 D 264 E 273

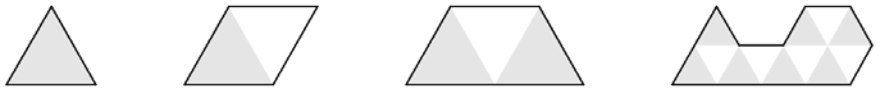
6. What is the sum of the digits of the largest 4-digit palindromic number which is divisible by 15? [Palindromic numbers read the same backwards and forwards, e.g. 7227.]

A 18 B 20 C 24 D 30 E 36

7. Given that $x + y + z = 1$, $x + y - z = 2$ and $x - y - z = 3$, what is the value of xyz?

A -2 B $-\frac{1}{2}$ C 0 D $\frac{1}{2}$ E 2

8. The diagrams below show four types of tile, each of which is made up of one or more equilateral triangles. For how many of these types of tile can we place three identical copies of the tile together, without gaps or overlaps, to make an equilateral triangle?



A 0 B 1 C 2 D 3 E 4

9. Pierre said, "Just one of us is telling the truth". Qadr said, "What Pierre says is not true".
Ratna said, "What Qadr says is not true". Sven said, "What Ratna says is not true".
Tanya said, "What Sven says is not true".

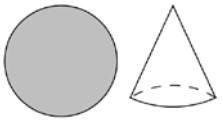
How many of them were telling the truth?

A 0 B 1 C 2 D 3 E 4

10. Let N be the smallest positive integer whose digits add up to 2012. What is the first digit of $N+1$?

A 2 B 3 C 4 D 5 E 6

11. Coco is making clown hats from a circular piece of cardboard. The circumference of the base of each hat equals its slant height, which in turn is equal to the radius of the piece of cardboard. What is the maximum number of hats that Coco can make from the piece of cardboard?



A 3 B 4 C 5 D 6 E 7

12. The number 3 can be expressed as the sum of one or more positive integers in four different ways:

$3; \quad 1+2; \quad 2+1; \quad 1+1+1.$

In how many ways can the number 5 be so expressed?

A 8 B 10 C 12 D 14 E 16

13. A cube is placed with one face on square 1 in the maze shown, so that it completely covers the square with no overlap. The upper face of the cube is covered in wet paint. The cube is 'rolled' around the maze, rotating about an edge each time, until it reaches square 25. It leaves paint on all of the squares on which the painted face lands, but on no others. The cube is removed on reaching the square 25. What is the sum of the numbers on the squares which are now marked with paint?

5	6	7	8	9
4	19	20	21	10
3	18	25	22	11
2	17	24	23	12
1	16	15	14	13

A 78 B 80 C 82 D 169 E 625

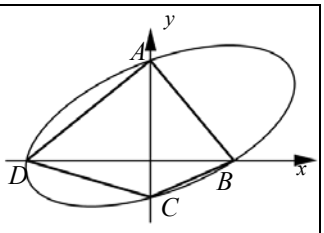
14. Six students who share a house all speak exactly two languages. Helga speaks only English and German; Ina speaks only German and Spanish; Jean-Pierre speaks only French and Spanish; Karim speaks only German and French; Lionel speaks only French and English whilst Mary speaks only Spanish and English. If two of the students are chosen at random, what is the probability that they speak a common language?

A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

15. Professor Rosseforp runs to work every day. On Thursday he ran 10% faster than his usual average speed. As a result, his journey time was reduced by x minutes. How many minutes did the journey take on Wednesday?

A $11x$ B $10x$ C $9x$ D $8x$ E $5x$

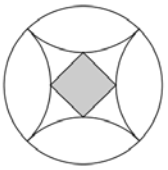
16. The diagram shows the ellipse whose equation is $x^2 + y^2 - xy + x - 4y = 12$. The curve cuts the y-axis at points A and C and cuts the x-axis at points B and D.



What is the area of the inscribed quadrilateral $ABCD$?

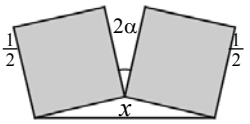
A 28 B 36 C 42 D 48 E 56

17. The diagram shows a pattern found on a floor tile in the cathedral in Spoleto, Umbria. A circle of radius 1 surrounds four quarter circles, also of radius 1, which enclose a square. The pattern has four axes of symmetry. What is the side length of the square?



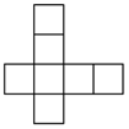
A $\frac{1}{\sqrt{2}}$ B $2-\sqrt{2}$ C $\frac{1}{\sqrt{3}}$ D $\frac{1}{2}$ E $\sqrt{2}-1$

18. The diagram shows two squares, with sides of length $\frac{1}{2}$, inclined at an angle 2α to one another. What is the value of x?



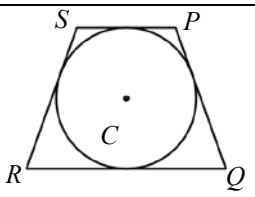
A $\cos\alpha$ B $\dfrac{1}{\cos\alpha}$ C $\sin\alpha$ D $\dfrac{1}{\sin\alpha}$ E $\tan\alpha$

19. The numbers 2, 3, 4, 5, 6, 7, 8 are to be placed, one per square, in the diagram shown so that the sum of the four numbers in the horizontal row equals 21 and the sum of the four numbers in the vertical column also equals 21. In how many different ways can this be done?



A 0 B 2 C 36 D 48 E 72

20. In trapezium $PQRS$, $SR = PQ = 25$ cm and SP is parallel to RQ. All four sides of $PQRS$ are tangent to a circle with centre C. The area of the trapezium is 600 cm^2. What is the radius of the circle?

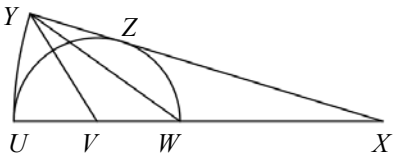


A 7.5cm B 8cm C 9cm D 10cm E 12cm

21. Which of the following numbers does *not* have a square root in the form $x + y\sqrt{2}$, where x and y are positive integers?

A $17 + 12\sqrt{2}$ B $22 + 12\sqrt{2}$ C $38 + 12\sqrt{2}$ D $54 + 12\sqrt{2}$ E $73 + 12\sqrt{2}$

22. A semicircle of radius r is drawn with centre V and diameter UW. The line UW is then extended to the point X, such that the UW and WX are of equal length. An arc of the circle with centre X and radius $4r$ is then drawn so that the line XY is a tangent to the semicircle at Z, as shown. What, in terms of r, is the area of triangle YVW?

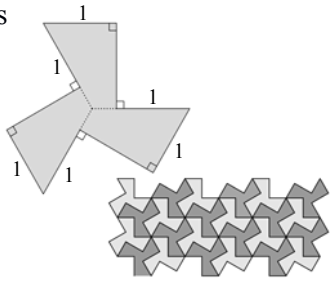


A $\frac{4r^2}{9}$ B $\frac{2r^2}{3}$ C r^2 D $\frac{4r^2}{3}$ E $2r^2$

23. Tom and Geri have a competition. Initially, each player has one attempt at hitting a target. If one player hits the target and the other does not then the successful player wins. If both players hit the target, or if both players miss the target, then each has another attempt, with the same rules applying. If the probability of Tom hitting the target is always $\frac{4}{5}$ and the probability of Geri hitting the target is always $\frac{2}{3}$, what is the probability that Tom wins the competition?

A $\frac{4}{15}$ B $\frac{8}{15}$ C $\frac{2}{3}$ D $\frac{4}{5}$ E $\frac{13}{15}$

24. The top diagram on the right shows a shape that tiles the plane, as shown in the lower diagram. The tile has nine sides, six of which have length 1. It may be divided into three congruent quadrilaterals as shown. What is the area of the tile?



A $\frac{1+2\sqrt{3}}{2}$ B $\frac{4\sqrt{3}}{3}$ C $\sqrt{6}$ D $\frac{3+4\sqrt{3}}{4}$ E $\frac{3\sqrt{3}}{2}$

25. How many distinct pairs (x, y) of real numbers satisfy the equation $(x+y)^2 = (x+4)(y-4)$?

A 0 B 1 C 2 D 3 E 4

UK SENIOR MATHEMATICAL CHALLENGE

November 8th 2011

1. Which of the numbers below is not a whole number?

A $\frac{2011+0}{1}$ B $\frac{2011+1}{2}$ C $\frac{2011+2}{3}$ D $\frac{2011+3}{4}$ E $\frac{2011+4}{5}$

2. Jack and Jill went up the hill to fetch a pail of water. Having filled the pail to the full, Jack fell down spilling $\frac{2}{3}$ of the water, before Jill caught the pail. She then tumbled down the hill, spilling $\frac{2}{5}$ of the remainder.

What fraction of the pail does the remaining water fill?

A $\frac{11}{15}$ B $\frac{1}{3}$ C $\frac{4}{15}$ D $\frac{1}{5}$ E $\frac{1}{15}$

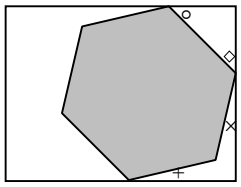
3. The robot *Lumber9* moves along the number line. *Lumber9* starts at 0, takes 1 step forward (to 1), then 2 steps backwards (to -1), then 3 steps forward, 4 steps back, and so on, moving alternately forwards and backwards, one more step each time. At what number is *Lumber9* after 2011 steps?

A 1006 B 27 C 11 D 0 E -18

4. What is the last digit of 3^{2011}?

A 1 B 3 C 5 D 7 E 9

5. The diagram shows a regular hexagon inside a rectangle.
What is the sum of the four marked angles?



A 90^0 B 120^0 C 150^0 D 180^0 E 210^0

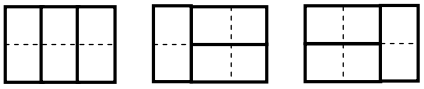
6. Granny and her granddaughter Gill both had their birthday yesterday. Today, Granny's age in years is an even number and 15 times that of Gill. In 4 year's time Granny's age in years will be the square of Gill's age in years. How many years older than Gill is Granny today?

A 42 B 49 C 56 D 60 E 64

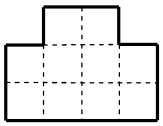
7. Two sides of a triangle have lengths 4cm and 5cm. The third side has length x cm, where x is a positive integer. How many different values can x have?

A 4 B 5 C 6 D 7 E 8

8. A 2×3 grid of squares can be divided into 1×2 rectangles in three different ways.



How many ways are there of dividing the bottom shape into 1×2 rectangles?



A 1 B 4 C 6 D 7 E 8

9. Sam has a large collection of $1 \times 1 \times 1$ cubes, each of which is either red or yellow. Sam makes a $3 \times 3 \times 3$ block from twenty-seven cubes, so that no cubes of the same colour meet face-to-face. What is the difference between the largest number of red cubes that Sam can use and the smallest number?

A 0 B 1 C 2 D 3 E 4

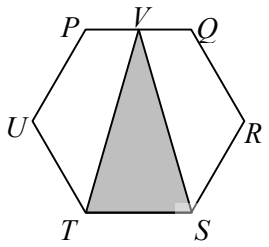
10. A triangle has two edges of length 5. What length should be chosen for the third edge of the triangle so as to maximize the area within the triangle?

A 5 B 6 C $5\sqrt{2}$ D 8 E $5\sqrt{3}$

11. *PQRSTU* is a regular hexagon, and *V* is the midpoint of *PQ*.

What fraction of the area of *PQRSTU* is the area of triangle *STV*?

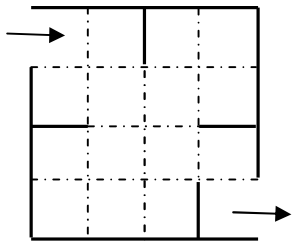
A $\frac{1}{4}$ B $\frac{2}{15}$ C $\frac{1}{3}$ D $\frac{2}{5}$ E $\frac{5}{12}$



12. The *primorial* of a number is the product of all the prime numbers less than or equal to that number. For example, the primorial of 6 is $2 \times 3 \times 5 = 30$. How many different whole numbers have a primorial of 210?

A 1 B 2 C 3 D 4 E 5

13. The diagram represents a maze. Given that you can only move horizontally and vertically and are not allowed to revisit a square, how many different routes are there through the maze?



A 16 B 12 C 10 D 8 E 6

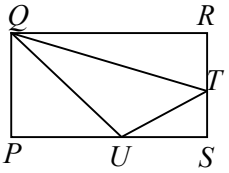
14. An equilateral triangle of side length 4 cm is divided up into smaller equilateral triangles, all of which have side length equal to a whole number of centimetres. Which of the following cannot be the number of smaller triangles obtained?

A 4 B 8 C 12 D 13 E 16

15. The equation $x^2 + ax + b = 0$, where a and b are different, has solutions $x = a$ and $x = b$.
How many such equations are there?

A 0 B 1 C 3 D 4 E an infinity

16. $PQRS$ is a rectangle. The area of triangle QRT is $\frac{1}{5}$ of the area of $PQRS$, and the area of triangle TSU is $\frac{1}{8}$ of the area of $PQRS$. What fraction of the area of rectangle $PQRS$ is the area of the triangle QTU?



A $\frac{27}{40}$ B $\frac{21}{40}$ C $\frac{1}{2}$ D $\frac{19}{40}$ E $\frac{23}{60}$

17. Jamie conducted a survey on the food preferences of pupils at a school and discovered that 70% of the pupils like pears, 75% like oranges, 80% like bananas and 85% like apples. What is the smallest possible percentage of pupils who like all four of these fruits?

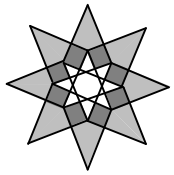
A at least 10% B at least 15% C at least 20% D at least 25% E at least 70%

18. Two numbers x and y are such that $x + y = 20$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$.

What is the value of $x^2y + xy^2$?

A 80 B 200 C 400 D 640 E 800

19. The diagram shows a small regular octagram (an eight-sided star) surrounded by eight squares (dark grey) and eight kites (light grey) to make a large regular octagram. Each square has area 1. What is the area of one of the light grey kites?



A 2 B $\sqrt{2}+1$ C $\frac{21}{8}$ D $4\sqrt{2}-3$ E $\frac{11}{4}$

20. Positive integers x and y satisfy the equation $\sqrt{x} - \sqrt{11} = \sqrt{y}$.

What is the maximum possible value of $\frac{x}{y}$?

A 2 B 4 C 8 D 11 E 44

21. Each of the Four Musketeers made a statement about the four of them, as follows:

d'Artagnan: "Exactly one is lying."

Athos: "Exactly two of us are lying."

Porthos: "An odd number of us is lying."

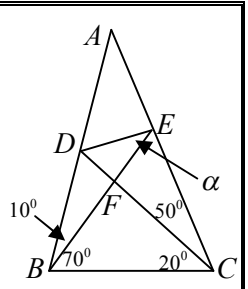
Aramis: "An even number of us is lying."

How many of them were lying (with the others telling the truth)?

A one B one or two C two or three D three E four

22. In the diagram, $\angle ABE = 10^0$; $\angle EBC = 70^0$; $\angle ACD = 50^0$; $\angle DCB = 20^0$; $\angle DEF = \alpha$.

Which of the following is equal to $\tan\alpha$?



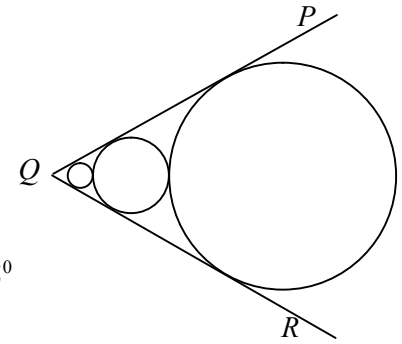
A $\dfrac{\tan 10^0 \tan 20^0}{\tan 50^0}$ B $\dfrac{\tan 10^0 \tan 20^0}{\tan 70^0}$ C $\dfrac{\tan 10^0 \tan 50^0}{\tan 70^0}$

D $\dfrac{\tan 20^0 \tan 50^0}{\tan 70^0}$ E $\dfrac{\tan 10^0 \tan 70^0}{\tan 50^0}$

23. What is the minimum value of $x^2 + y^2 + 2xy + 6x + 6y + 4$?

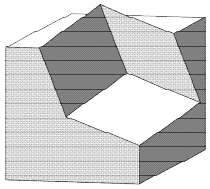
A -7 B -5 C -4 D -1 E 4

24. Three circles and the lines PQ and QR touch as shown. The distance between the centres of the smallest and biggest circles is 16 times the radius of the smallest circle. What is the size of $\angle PQR$?



A 45^0 B 60^0 C 75^0 D 90^0 E 135^0

25. A solid sculpture consists of a $4 \times 4 \times 4$ cube with a $3 \times 3 \times 3$ cube sticking out as shown. Three vertices of the smaller cube lie on edges of the larger cube, the same distance along each edge. What is the total volume of the sculpture?



A 79 B 81 C 82 D 84 E 85