UK Intermediate

Mathematical Challenges

2014 to 2016

Bromsgrove International School Thailand

UK INTERMEDIATE MATHEMATICAL CHALLENGE

February 6th 2014

Supported by

Institute and Faculty of Actuaries

1. What is 25% of $\frac{3}{4}$?

A $\frac{3}{16}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D 1 E 3

2. Which is the smallest positive integer for which all these are true?

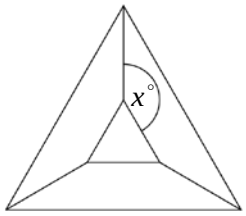
(i) It is odd.
(ii) It is not prime.
(iii) The next largest odd integer is not prime.

A 9 B 15 C 21 D 25 E 33

3. An equilateral triangle is placed inside a larger equilateral triangle so that the diagram has three lines of symmetry.

What is the value of x?

A 100 B 110 C 120 D 130 E 150



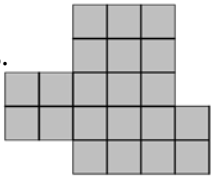
4. You are given that m is an even integer and n is an odd integer. Which of these is an odd integer?

A $3m + 4n$ B $5mn$ C $(m + 3n)^2$ D m^3n^3 E $5m + 6n$

5. A ship's bell is struck every half hour, starting with one bell at 0030, two bells (meaning the bell is struck twice) at 0100, three bells at 0130 until the cycle is complete with eight bells at 0400. The cycle then starts again with one bell at 0430, two bells at 0500 and so on. What is the total number of times the bell is struck between 0015 on one day and 0015 on the following day?

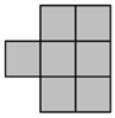
A 24 B 48 C 108 D 144 E 216

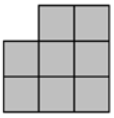
6. The shape shown on the right was assembled from three identical copies of one of the smaller shapes below, without gaps or overlaps.



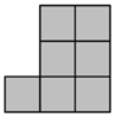
Which smaller shape was used?

A 

B 

C 

D 

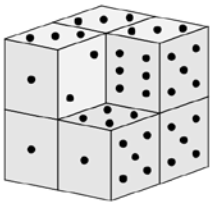
E 

7. Just one positive integer has exactly 8 factors including 6 and 15.

What is the integer?

A 21 B 30 C 45 D 60 E 90

8. A large cube is made by stacking eight dice. The diagram shows the result, except that one of the dice is missing. Each die has faces with 1, 2, 3, 4, 5 and 6 pips and the total number of pips on opposite faces is 7. When two dice are placed face to face, the matching faces must have the same number of pips.



What could the missing die look like?

A 

B 

C 

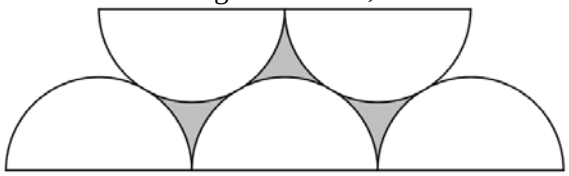
D 

E 

9. At the age of twenty-six, Gill has passed her driving test and bought a car. Her car uses p litres of petrol per 100 km travelled. How many litres of petrol would be required for a journey of d km?

A $\frac{pd}{100}$ B $\frac{100p}{d}$ C $\frac{100d}{p}$ D $\frac{100}{pd}$ E $\frac{p}{100d}$

10. The diagram shows five touching semicircles, each with radius 2.



What is the length of the perimeter of the shaded shape?

A 5π B 6π C 7π D 8π E 9π

11. Not all characters in the Woodentops series tell the truth. When Mr Plod asked them, "How many people are there in the Woodentops family?", four of them replied as follows:

Jenny: "An even number." Willie: "An odd number." Sam: "A prime number."

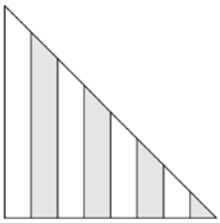
Mrs Scrubitt: "A number which is the product of two integers greater than one."

How many of these four were telling the truth?

A 0 B 1 C 2 D 3 E 4

12. The diagram shows an isosceles right-angled triangle divided into strips of equal width. Four of the strips are shaded.

What fraction of the area of the triangle is shaded?



A $\frac{11}{32}$ B $\frac{3}{8}$ C $\frac{13}{32}$ D $\frac{7}{16}$ E $\frac{15}{32}$

13. How many numbers can be written as a sum of two different positive integers each at most 100?

A 100 B 197 C 198 D 199 E 200

14. This year the *Tour de France* starts in Leeds on 5 July. Last year, the total length of the *Tour* was 3404 km and the winner, Chris Froome, took a total time of 83 hours 56 minutes 40 seconds to cover this distance. Which of these is closest to his average speed over the whole event?

A 32km/h B 40 km/h C 48 km/h D 56 km/h E 64 km/h

15. Zac halves a certain number and then adds 8 to the result. He finds that he obtains the same answer if he doubles his original number and then subtracts 8 from the result.

What is Zac's original number?

A $8\frac{2}{3}$ B $9\frac{1}{3}$ C $9\frac{2}{3}$ D $10\frac{1}{3}$ E $10\frac{2}{3}$

16. The base of a triangle is increased by 25% but the area of the triangle is unchanged.

By what percentage is the corresponding perpendicular height decreased?

A $12\frac{1}{2}\%$ B 16% C 20% D 25% E 50%

17. How many weeks are there in $8\times7\times6\times5\times4\times3\times2\times1$ minutes?

A 1 B 2 C 3 D 4 E 5

18. Consider looking from the origin (0,0) towards all the points (m,n), where each of m and n is an integer. Some points are *hidden*, because they are directly in line with another nearer point. For example, (2,2) is hidden by (1,1).

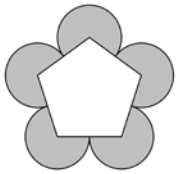
How many of the points (6,2), (6,3), (6,4) and (6,5) are *not* hidden points?

A 0 B 1 C 2 D 3 E 4

19. Suppose that $8^m = 27$. What is the value of 4^m?

A 3 B 4 C 9 D 13.5 E there is no such m

20. The diagram shows a regular pentagon and five circular arcs. The sides of the pentagon have length 4. The centre of each arc is a vertex of the pentagon, and the ends of the arc are the midpoints of the two adjacent edges.



What is the total shaded area?

A 8π B 10π C 12π D 14π E 16π

21. In King Arthur's jousting tournament, each of the several competing knights receives 17 points for every bout he enters. The winner of each bout receives an extra 3 points. At the end of the tournament, the Black Knight has exactly one more point than the Red Knight.

What is the smallest number of bouts that the Black Knight could have entered?

A 3 B 4 C 5 D 6 E 7

22. The positive integers a, b and c are all different. None of them is a square but all the products ab, ac and bc are squares. What is the least value that $a+b+c$ can take?

A 14 B 28 C 42 D 56 E 70

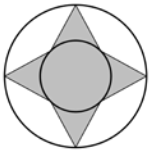
23. A sector of a disc is removed by making two straight cuts from the circumference to the centre. The perimeter of the sector has the same length as the circumference of the original disc. What fraction of the area of the disc is removed?

A $\frac{\pi-1}{\pi}$ B $\frac{1}{\pi}$ C $\frac{\pi}{360}$ D $\frac{1}{3}$ E $\frac{1}{2}$

24. How many 4-digit integers (from 1000 to 9999) have at least one digit repeated?

A 62×72 B 52×72 C 52×82 D 42×82 E 42×92

25. The diagram shows two concentric circles with radii of 1 and 2 units, together with a shaded octagon, all of whose sides are equal.



What is the length of the perimeter of the octagon?

A $8\sqrt{2}$ B $8\sqrt{3}$ C $8\sqrt{3}\pi$ D $2\sqrt{5+2\sqrt{2}}$ E $8\sqrt{5-2\sqrt{2}}$

Intermediate Mathematical Challenge

Thursday 5th February 2015

Organised by the United Kingdom Mathematics Trust

supported by Institute and Faculty of Actuaries

1. What is the value of $1 - 0.2 + 0.03 - 0.004$?

A 0.826 B 0.834 C 0.926 D 1.226 E 1.234

2. Last year, Australian Suzy Walsham won the annual women's race up the 1576 steps of the Empire State Building in New York for a record fifth time. Her winning time was 11 minutes 57 seconds.

Approximately how many steps did she climb per minute?

A 13 B 20 C 80 D 100 E 130

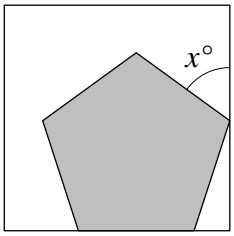
3. What is a half of a third, plus a third of a quarter, plus a quarter of a fifth?

A $\frac{1}{1440}$ B $\frac{3}{38}$ C $\frac{1}{30}$ D $\frac{1}{3}$ E $\frac{3}{10}$

4. The diagram shows a regular pentagon inside a square.

What is the value of x?

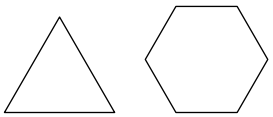
A 48 B 51 C 54 D 60 E 72



5. Which of the following numbers is not a square?

A 1^6 B 2^5 C 3^4 D 4^3 E 5^2

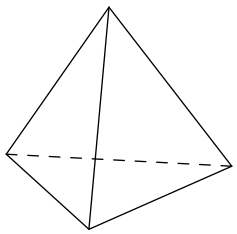
6. The equilateral triangle and regular hexagon shown have perimeters of the same length.



What is the ratio of the area of the triangle to the area of the hexagon?

A 5 : 6 B 4 : 5 C 3 : 4 D 2 : 3 E 1 : 1

7. A tetrahedron is a solid figure which has four faces, all of which are triangles.



What is the product of the number of edges and the number of vertices of the tetrahedron?

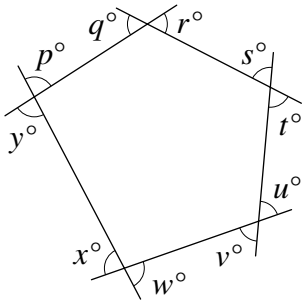
A 8 B 10 C 12 D 18 E 24

8. How many two-digit squares differ by 1 from a multiple of 10?

A 1 B 2 C 3 D 4 E 5

9. What is the value of $p + q + r + s + t + u + v + w + x + y$ in the diagram?

A 540 B 720 C 900 D 1080
E 1440



10. What is the remainder when $2^2 \times 3^3 \times 5^5 \times 7^7$ is divided by 8?

A 2 B 3 C 4 D 5 E 7

11. Three different positive integers have a mean of 7.

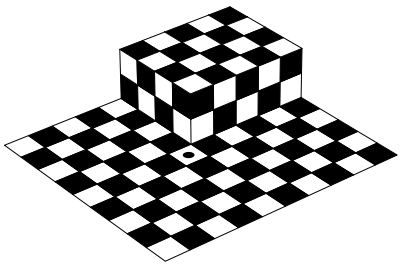
What is the largest positive integer that could be one of them?

A 15 B 16 C 17 D 18 E 19

12. An ant is on the square marked with a black dot. The ant moves across an edge from one square to an adjacent square four times and then stops.

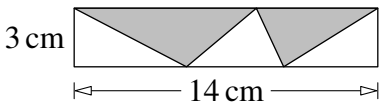
How many of the possible finishing squares are black?

A 0 B 2 C 4 D 6 E 8



13. What is the area of the shaded region in the rectangle?

A $21\,\text{cm}^2$ B $22\,\text{cm}^2$ C $23\,\text{cm}^2$
D $24\,\text{cm}^2$ E more information needed

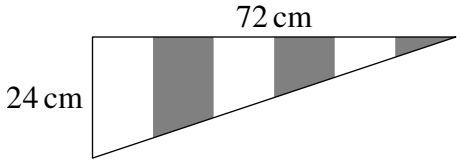


14. In a sequence, each term after the first two terms is the mean of all the terms which come before that term. The first term is 8 and the tenth term is 26.

What is the second term?

A 17 B 18 C 44 D 52 E 68

15. A flag is in the shape of a right-angled triangle, as shown, with the horizontal and vertical sides being of length 72 cm and 24 cm respectively. The flag is divided into 6 vertical stripes of equal width.



What, in cm^2, is the difference between the areas of any two adjacent stripes?

A 96 B 72 C 48 D 32 E 24

16. You are asked to choose two positive integers, m and n with $m > n$, so that as many as possible of the expressions $m + n$, $m - n$, $m \times n$ and $m \div n$ have values that are prime.

When you do this correctly, how many of these four expressions have values that are prime?

A 0 B 1 C 2 D 3 E 4

17. The football shown is made by sewing together 12 black pentagonal panels and 20 white hexagonal panels. There is a join wherever two panels meet along an edge.

How many joins are there?

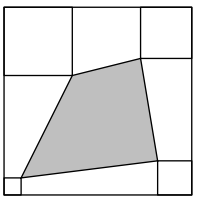
A 20 B 32 C 60 D 90 E 180

18. The total weight of a box, 20 plates and 30 cups is 4.8 kg. The total weight of the box, 40 plates and 50 cups is 8.4 kg.

What is the total weight of the box, 10 plates and 20 cups?

A 3 kg B 3.2 kg C 3.6 kg D 4 kg E 4.2 kg

19. The figure shows four smaller squares in the corners of a large square. The smaller squares have sides of length 1 cm, 2 cm, 3 cm and 4 cm (in anticlockwise order) and the sides of the large square have length 11 cm.



What is the area of the shaded quadrilateral?

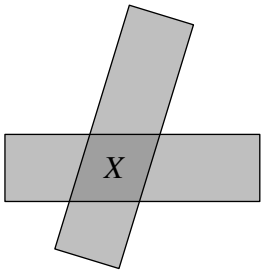
A $35\,\text{cm}^2$ B $36\,\text{cm}^2$ C $37\,\text{cm}^2$ D $38\,\text{cm}^2$ E $39\,\text{cm}^2$

20. A voucher code is made up of four characters. The first is a letter: V, X or P. The second and third are different digits. The fourth is the units digit of the sum of the second and third digits.

How many different voucher codes like this are there?

A 180 B 243 C 270 D 300 E 2700

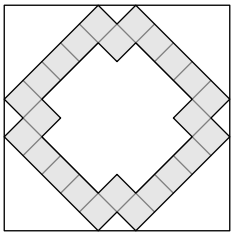
21. A rectangle is placed obliquely on top of an identical rectangle, as shown. The area X of the overlapping region (shaded more darkly) is one eighth of the total shaded area.



What fraction of the area of one rectangle is X?

A $\frac{1}{3}$ B $\frac{2}{7}$ C $\frac{1}{4}$ D $\frac{2}{9}$ E $\frac{1}{5}$

22. The diagram shows a shaded region inside a large square. The shaded region is divided into small squares.



What fraction of the area of the large square is shaded?

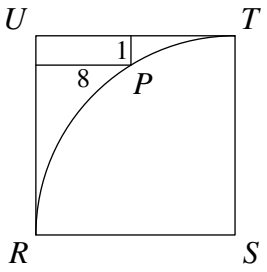
A $\frac{3}{10}$ B $\frac{1}{3}$ C $\frac{3}{8}$ D $\frac{2}{5}$ E $\frac{3}{7}$

23. There are 120 different ways of arranging the letters, U, K, M, I and C. All of these arrangements are listed in dictionary order, starting with CIKMU.

Which position in the list does UKIMC occupy?

A 110th B 112th C 114th D 116th E 118th

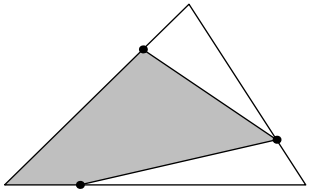
24. In square $RSTU$ a quarter-circle arc with centre S is drawn from T to R. A point P on this arc is 1 unit from TU and 8 units from RU.



What is the length of the side of square $RSTU$?

A 9 B 10 C 11 D 12 E 13

25. A point is marked one quarter of the way along each side of a triangle, as shown.



What fraction of the area of the triangle is shaded?

A $\frac{7}{16}$ B $\frac{1}{2}$ C $\frac{9}{16}$ D $\frac{5}{8}$ E $\frac{11}{16}$

2016

Intermediate Mathematical Challenge

Organised by the United Kingdom Mathematics Trust

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1. What is the value of $6102 - 2016$?

A 3994 B 4086 C 4096 D 4114 E 4994

2. Which of the following fractions is closest to 1?

A $\frac{7}{8}$ B $\frac{8}{7}$ C $\frac{9}{10}$ D $\frac{10}{11}$ E $\frac{11}{10}$

3. How many of these five expressions give answers which are *not* prime numbers?

$1^2 + 2^2$ $2^2 + 3^2$ $3^2 + 4^2$ $4^2 + 5^2$ $5^2 + 6^2$

A 0 B 1 C 2 D 3 E 4

4. Amrita is baking a cake today. She bakes a cake every fifth day.

How many days will it be before she next bakes a cake on a Thursday?

A 5 B 7 C 14 D 25 E 35

5. When travelling from London to Edinburgh by train, you pass a sign saying "Edinburgh 200 miles". Then, $3\frac{1}{2}$ miles later, you pass another sign saying "Half way between London and Edinburgh".

How many miles is it by train from London to Edinburgh?

A 393 B $396\frac{1}{2}$ C 400 D $403\frac{1}{2}$ E 407

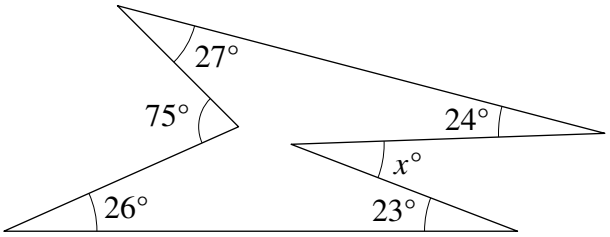
6. One third of the animals in Jacob's flock are goats, the rest are sheep. There are twelve more sheep than goats.

How many animals are there altogether in Jacob's flock?

A 12 B 24 C 36 D 48 E 60

7. In the diagram, what is the value of x?

A 23 B 24 C 25
D 26 E 27



8. What is the value of $2.017 \times 2016 - 10.16 \times 201.7$?

A 2.016 B 2.017 C 20.16 D 2016 E 2017

9. The world's fastest tortoise is acknowledged to be a leopard tortoise from County Durham called Bertie. In July 2014, Bertie sprinted along a 5.5 m long track in an astonishing 19.6 seconds.

What was Bertie's approximate average speed in km per hour?

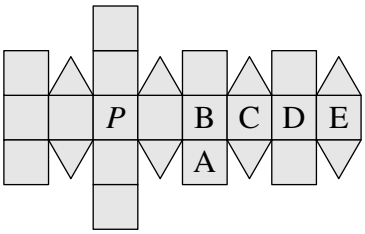
A 0.1 B 0.5 C 1 D 5 E 10

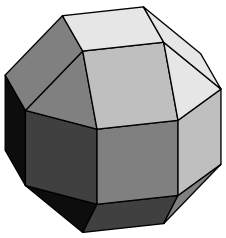
10. The angles of a quadrilateral taken in order are $x°$, $5x°$, $2x°$ and $4x°$.

Which of the following is the quadrilateral?

A kite B parallelogram C rhombus D arrowhead
E trapezium

11. The net shown consists of squares and equilateral triangles. The net is folded to form a rhombicuboctahedron, as shown.





When the face marked P is placed face down on a table, which face will be facing up?

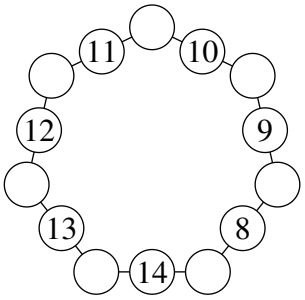
A B C D E

12. The sum of two numbers a and b is 7 and the difference between them is 2.

What is the value of $a \times b$?

A $8\frac{1}{4}$ B $9\frac{1}{4}$ C $10\frac{1}{4}$ D $11\frac{1}{4}$ E $12\frac{1}{4}$

13. The diagram shows a heptagon with a line of three circles on each side. Each circle is to contain exactly one number. The numbers 8 to 14 are distributed as shown and the numbers 1 to 7 are to be distributed to the remaining circles. The total of the numbers in each of the lines of three circles is to be the same.



What is this total?

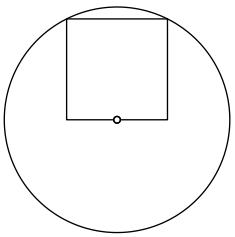
A 18 B 19 C 20 D 21 E 22

14. Tegwen has the same number of brothers as she has sisters. Each one of her brothers has 50% more sisters than brothers.

How many children are in Tegwen's family?

A 5 B 7 C 9 D 11 E 13

15. The circle has radius 1 cm. Two vertices of the square lie on the circle. One edge of the square goes through the centre of the circle, as shown.



What is the area of the square?

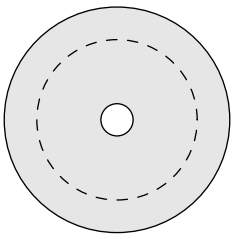
A $\frac{4}{5}\,\text{cm}^2$ B $\frac{\pi}{5}\,\text{cm}^2$ C $1\,\text{cm}^2$ D $\frac{\pi}{4}\,\text{cm}^2$ E $\frac{5}{4}\,\text{cm}^2$

16. How many of the following positive integers are divisible by 24?

$2^2 \times 3^2 \times 5^2 \times 7^3$ $\quad$ $2^2 \times 3^2 \times 5^3 \times 7^2$ $\quad$ $2^2 \times 3^3 \times 5^2 \times 7^2$ $\quad$ $2^3 \times 3^2 \times 5^2 \times 7^2$

A 0 $\quad$ B 1 $\quad$ C 2 $\quad$ D 3 $\quad$ E 4

17. The shaded region in the diagram, bounded by two concentric circles, is called an *annulus*. The circles have radii 2 cm and 14 cm.



The dashed circle divides the area of this annulus into two equal areas. What is its radius?

A 9 cm B 10 cm C 11 cm D 12 cm
E 13 cm

18. The sum of the areas of the squares on the sides of a right-angled isosceles triangle is $72\,\text{cm}^2$.

What is the area of the triangle?

A $6\,\text{cm}^2$ B $8\,\text{cm}^2$ C $9\,\text{cm}^2$ D $12\,\text{cm}^2$ E $18\,\text{cm}^2$

19. A list of positive integers has a median of 8, a mode of 9 and a mean of 10.

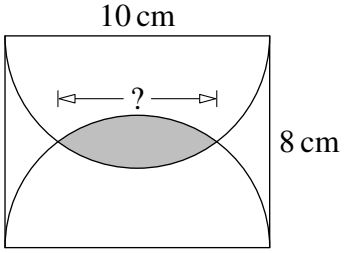
What is the smallest possible number of integers in the list?

A 5 B 6 C 7 D 8 E 9

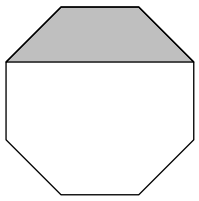
20. Two semicircles are drawn in a rectangle as shown.

What is the width of the overlap of the two semicircles?

A 3 cm B 4 cm C 5 cm D 6 cm
E 7 cm



21. The diagram shows a regular octagon.



What is the ratio of the area of the shaded trapezium to the area of the whole octagon?

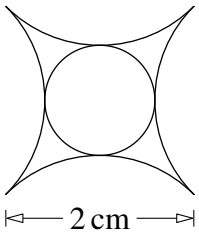
A 1 : 4 B 5 : 16 C 1 : 3 D $\sqrt{2}$: 2
E 3 : 8

22. In a particular group of people, some always tell the truth, the rest always lie. There are 2016 in the group. One day, the group is sitting in a circle. Each person in the group says, "Both the person on my left and the person on my right are liars."

What is the difference between the largest and smallest number of people who could be telling the truth?

A 0 B 72 C 126 D 288 E 336

23. A Saxon silver penny, from the reign of Ethelbert II in the eighth century, was sold in 2014 for £78 000. A design on the coin depicts a circle surrounded by four equal arcs, each a quarter of a circle, as shown. The width of the design is 2 cm.



What is the radius of the small circle, in centimetres?

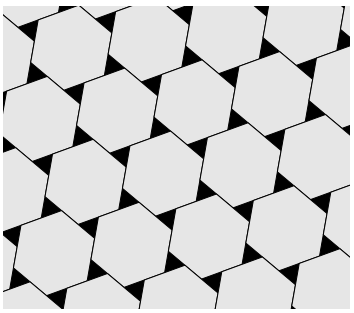
A $\frac{1}{2}$ B $2-\sqrt{2}$ C $\frac{1}{2}\sqrt{2}$ D $5-3\sqrt{2}$ E $2\sqrt{2}-2$

24. Every day, Aimee goes up an escalator on her journey to work. If she stands still, it takes her 60 seconds to travel from the bottom to the top. One day the escalator was broken so she had to walk up it. This took her 90 seconds.

How many seconds would it take her to travel up the escalator if she walked up at the same speed as before while it was working?

A 30 B 32 C 36 D 45 E 75

25. The tiling pattern shown uses two types of tile, regular hexagons and equilateral triangles, with the length of each side of the equilateral triangles equal to half the length of the sides of each side of the hexagons. A large number of tiles is used to cover a floor.



Which of the following is closest to the fraction of the floor that is shaded black?

A $\frac{1}{8}$ B $\frac{1}{10}$ C $\frac{1}{12}$ D $\frac{1}{13}$ E $\frac{1}{16}$